Mathematical modeling of energy flow in a geothermal reservoir

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The topic

The project involves the following:

- Calculation of fluid flow in porous media, according to Darcy's law.

$$\vec{q}_g = -\frac{\bar{\kappa}_r \kappa_g}{\mu_g}(\nabla p + \rho_m \vec{g}) \qquad \vec{q}_l = -\frac{\bar{\kappa}_r \kappa_l}{\mu_l}(\nabla p + \rho_m \vec{g})$$

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_g \vec{q}_g + \rho_l \vec{q}_l) = 0$$

- Calculation of energy balance

$$\phi \rho_m \frac{\partial h}{\partial t} + (1-\phi)\rho_r c_r \frac{\partial T_r}{\partial t} + \vec{q} \cdot \nabla h = \nabla \cdot (k_r \nabla T_r)$$

- Estimation of model parameters, based on data.

Basic assumptions

- The reservoir consists of different porous rock formations. The pores are filled with fluid, primarily water, which exists in liquid phase, gas phase, or as a two phase mixture. The solid rock material has elastic properties but does not move as the fluid itself.
- The reservoir is bounded by Earth's surface as well as outer boundaries within the crust. Conditions must be specified for selected variables everywhere on the boundary.
- All variables and parameters are functions of position in a three dimensional space, and in some cases parameters depend on some of the variables.
- The state of the reservoir generally varies with time, but in some cases a steady state solution can be found.

Motivation for the project

Various models exist, notably iTOUGH2, developed at Berkeley Labs. But, . . . the purpose of this work is to:

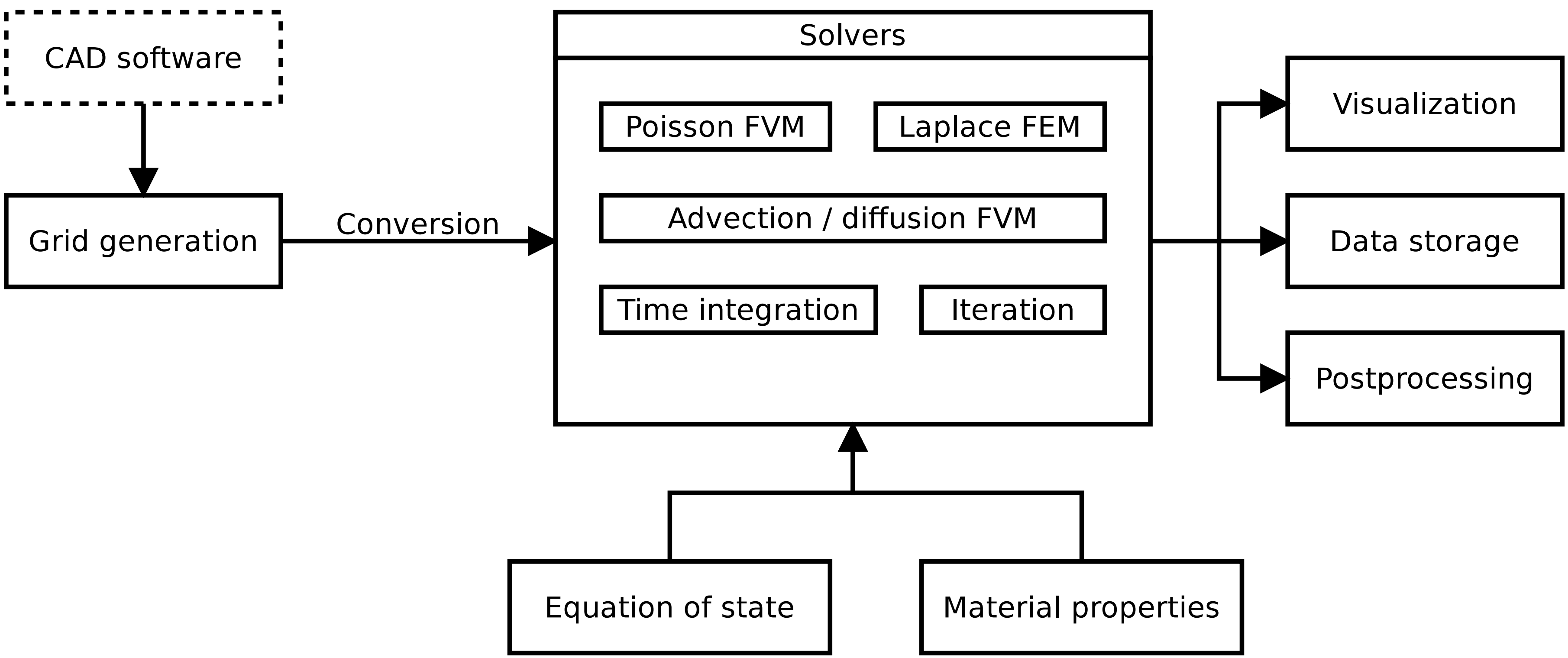
- Develop an Icelandic reservoir model in cooperation between University of Iceland, Reykjavík University and Icelandic GeoSurvey.
- Use the model or different parts of it for teaching and training students and specialist at the mentioned institutions.
- Involve people from different fields in the work such as geophysicists, engineers and mathematicians.
- Promote new ideas and methods in the modeling work, using state of the art methods from the literature.

The research group

- Assoc. prof. Elínborg I. Ólafsdóttir, UI
- Dept. man. Guðni Axelsson, IGS
- Ass. prof. Guðrún Sævarsdóttir, RU
- Assoc. prof. Halldór Pálsson, UI
- Assoc. prof. Jan Valdman, UI
- Prof. emer. Jónas Elíasson, UI
- PhD. stud. Lárus Þorvaldsson
- Prof. Ragnar Sigurðsson, UI
- Res. spec. Stefán I. Valdimarsson, UI

Software framework

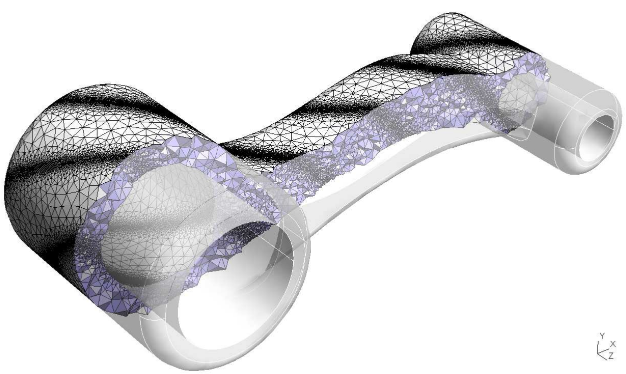
- Modular software will well defined interfaces.
- Focus on numerical mathematics and solvers.

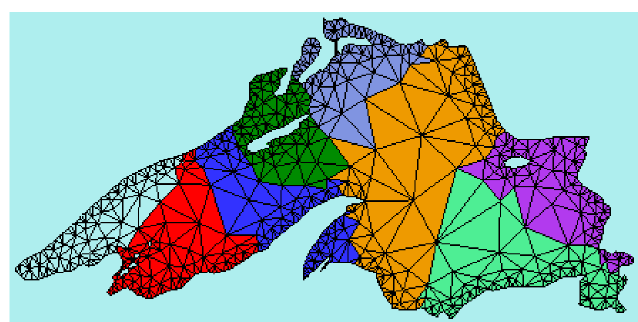


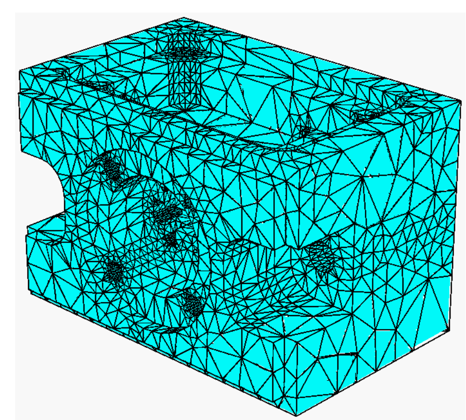
Mesh generation

Several high quality programs are available:

- GMSH (http://www.geuz.org/gmsh/), two and three dimensional, mainly trinangles and tetrahedra elements.
- Triangle (http://www.cs.cmu.edu/ quake/triangle.html), two dimensional Delaunay triangulator. Works well for FEM and FVM.
- TetGen (http://tetgen.berlios.de/), three dimensional tetrahedra generator. File interface similar to Triangle.







Discretization methods

Two methods are mostly used in practice:

- The finite element method (FEM). Has a solid mathematical background, but can be difficult to implement for advective problems.
- The finite volume method (FVM). Closely related to finite difference methods, based on conservation laws. Very common in problems involving fluid flow and advection.

Other alternatives are:

- Spectral methods, where the solution consists of high order functions (polynomials or trigonometric). Very accurate, but difficult to implement for complex domains.

Solution strategies

Discretization results in a large system of equations, possibly non-linear. Solution strategies involve

- Direct solution of linear equations.
- Iterative methods for linear equations, enhanced by preconditioners:
  - Incomplete factorization.
  - Fast methods based on FFT.
  - Multigrid methods, notably AMG.
- Non-linear iterations, e.g. Newton-Raphson with trust region or linesearch, or more specialized methods.
- Time stepping methods for unsteady problems.

Interpretation of results

Results are stored in a consistent manner. Postprocessing involves:

- Calculation of combined values, using e.g. Matlab or Octave.
- Visualization of field variables with:
  - the extensive graphical abilities of Matlab.
  - available programs, like ParaView (http://www.paraview.org/).
  - known commercial programs, e.g. ANSYS or Fluent.
- Comparison of results with other calculations.

Parameter estimation

Motivation:

- It is necessary to be able to estimate reservoir parameters, based on known data.
- Known as *inverse modeling* as an alternative to *parameter estimation*.
- A replication of the abilities of iTOUGH2.

Methods available for estimation:

- Standard methods, e.g. Levenberg-Marquardt.
- Robust methods with stochastic gradient search.
- Evolutionary programming for very difficult problems.

A simple case: Two dimensional flow

The Darcy equation for pressure variation

$$\nabla \cdot (\nabla \phi - \mathsf{Ra}\,\theta \vec{z}) = 0$$

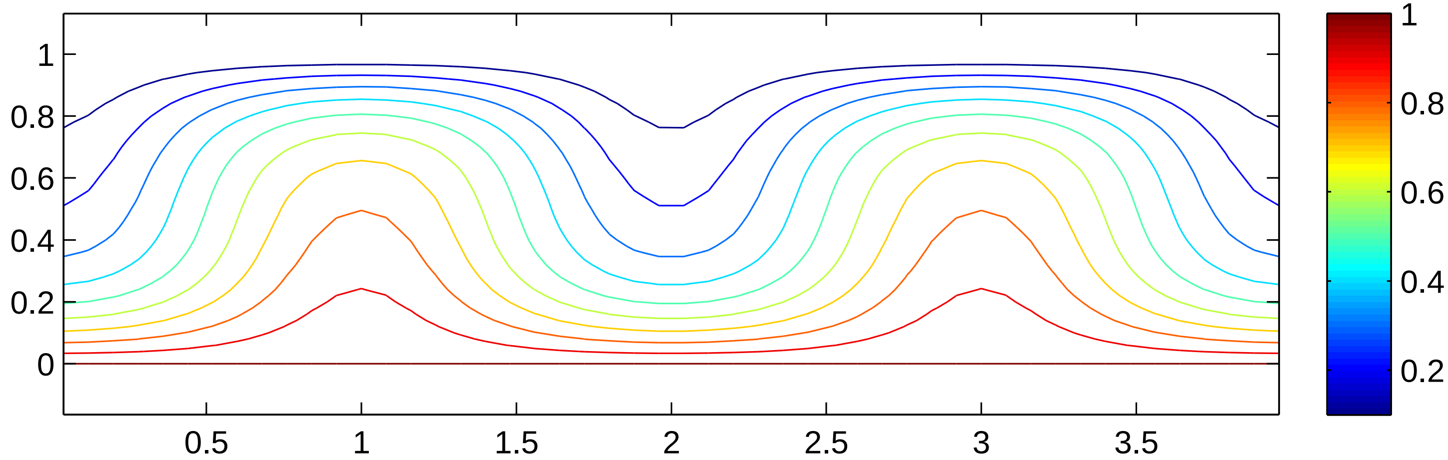
coupled with the advective-diffusive energy balance equation

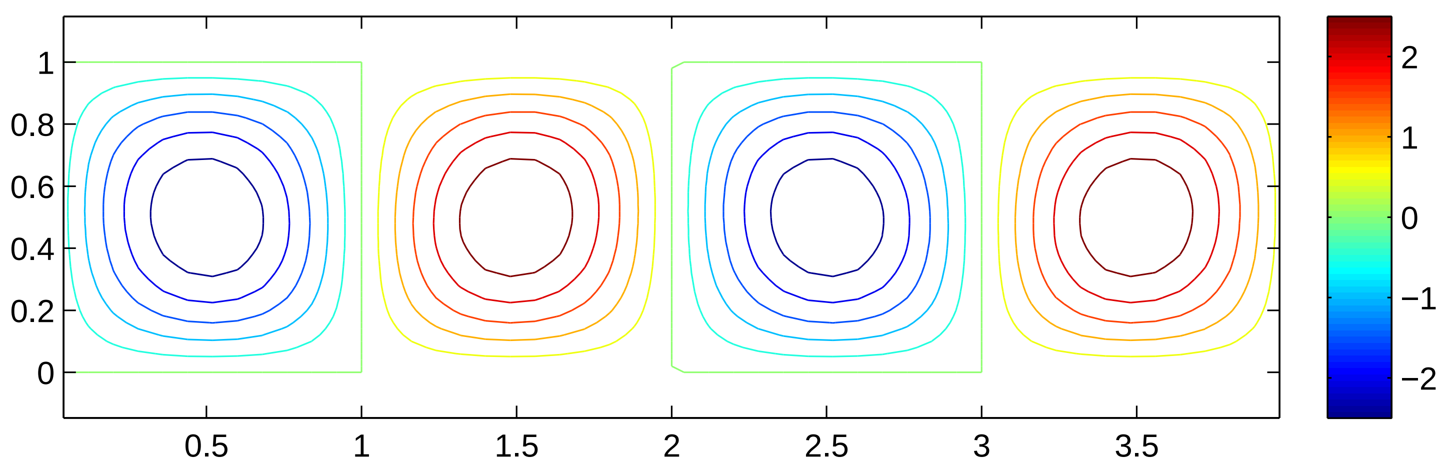
$$\frac{\partial \theta}{\partial \tau} = \nabla \cdot \big((\nabla \phi - \mathsf{Ra}\,\theta \vec{z})\,\theta + \nabla \theta \big)$$

where

$$\mathsf{Ra} = \frac{\rho^2 c g \beta (T_1 - T_0) \kappa L}{\mu k}$$

Results for Ra $= 60$





Conclusions

- An initiative has been started by UI, RU and IGS to develop their own geothermal reservoir model.
- The people involved are applied mathematicians, geophysicists and engineers.
- The purpose is to educate students and scientists and involve them into the inner workings and development of state of the art reservoir models.
- An emphasis will be put on using the newest findings in implementing solution methods, e.g. for solving large sets of equations and computation on clusters.
- Finally, the group hopes that the developed models will be used by research institutions and industry involved in utilization of geothermal energy.