Mathematical modeling of energy flow in a geothermal reservoir

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Objective of the project

To develop a model of two phase flow in reservoirs, based on a flexible existing framework of computational fluid dynamics software.

- Laminar, incompressible, single phase flow and energy transfer. Flow is quasi-steady, temperature is unsteady.
- Two phase flow without phase change, including a dynamic well bore flow model.
- Two phase flow with phase change, using a steady state solution for flow if appropriate, but dynamic energy calculations.
- Multiphase flow with phase change between water and steam and additional gas mixtures included.

Variables and equation of state

The state of the reservoir is given by pressure p, enthalpy h, density ρ and saturation α.

- A general equation of state is given as

$$\rho = f(p, h)$$

- A specific equation for compressible liquid can be written as

$$\rho = \rho_0 \exp\left(\frac{p - p_0}{E} - \frac{\beta(h - h_0)}{c_p}\right)$$

- Finally, the Boussinesq approximation is

$$\rho = \rho_0\big(1 - \beta(T - T_0)\big)$$

Conservation of mass

The continuity equation for a single phase is given in general form as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

The equation of state can be included in the time derivative, giving

$$\frac{\partial \rho}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial \rho}{\partial h}\frac{\partial h}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

In case of no phase change, saturation changes must be included, e.g.

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\vec{u}\alpha) = 0$$

Conservation of energy

For the fluid, the energy conservation is written as

$$\frac{\partial}{\partial t}(\rho h) + \nabla \cdot (\rho \vec{u} h) = \nabla \cdot \left(\frac{k}{c_p} \nabla h \right) + S$$

assuming no viscous heating. The source term S can include e.g. heat transfer coupling to the solid material. For the solid

$$\rho_s c_s \frac{\partial T}{\partial t} = \nabla \cdot (k_s \nabla T) + S_s$$

where the source term S_s is similar to the fluid source term, e.g. coupling between the two.

Velocity-pressure coupling

For most cases, the coupling can be expressed with the Darcy-Forchheimer relation

$$\nabla p + \rho\vec{g} = -\frac{\mu}{\bar{\kappa}}\vec{u} - \frac{\rho f}{2d}|\vec{u}|\vec{u}$$

where $\vec{u}$ is the superficial velocity. This formulation can e.g. be inserted into the conservation of mass for laminar liquid,

$$\frac{\rho}{E}\frac{\partial p}{\partial t} = \nabla \cdot \left(\frac{\rho\bar{\kappa}}{\mu}(\nabla p + \rho\vec{g})\right) + \frac{\beta\rho}{c_p}\frac{\partial h}{\partial t}$$

Dimensionless variables

Dimensionless temperature is defined as

$$\theta = \frac{T - T_0}{T_1 - T_0}$$

the dimensionless pressure as

$$\phi = \frac{\rho_0 c \kappa}{\mu k} \left(p + \rho_0 g L z\right)$$

and the dimensionless time as

$$\tau = \left(\frac{k}{\rho_0 c L^2}\right) t$$

The Darcy-Lapwood system

By introduction the dimensionless field variables the heat equation becomes

$$\frac{\partial \theta}{\partial \tau} = \nabla \cdot \big((\nabla \phi - \mathsf{Ra}\, \theta \vec{z})\, \theta + \nabla \theta \big)$$

and the continuity requirement is then

$$\nabla \cdot (\nabla \phi - \mathsf{Ra}\, \theta \vec{z}) = 0$$

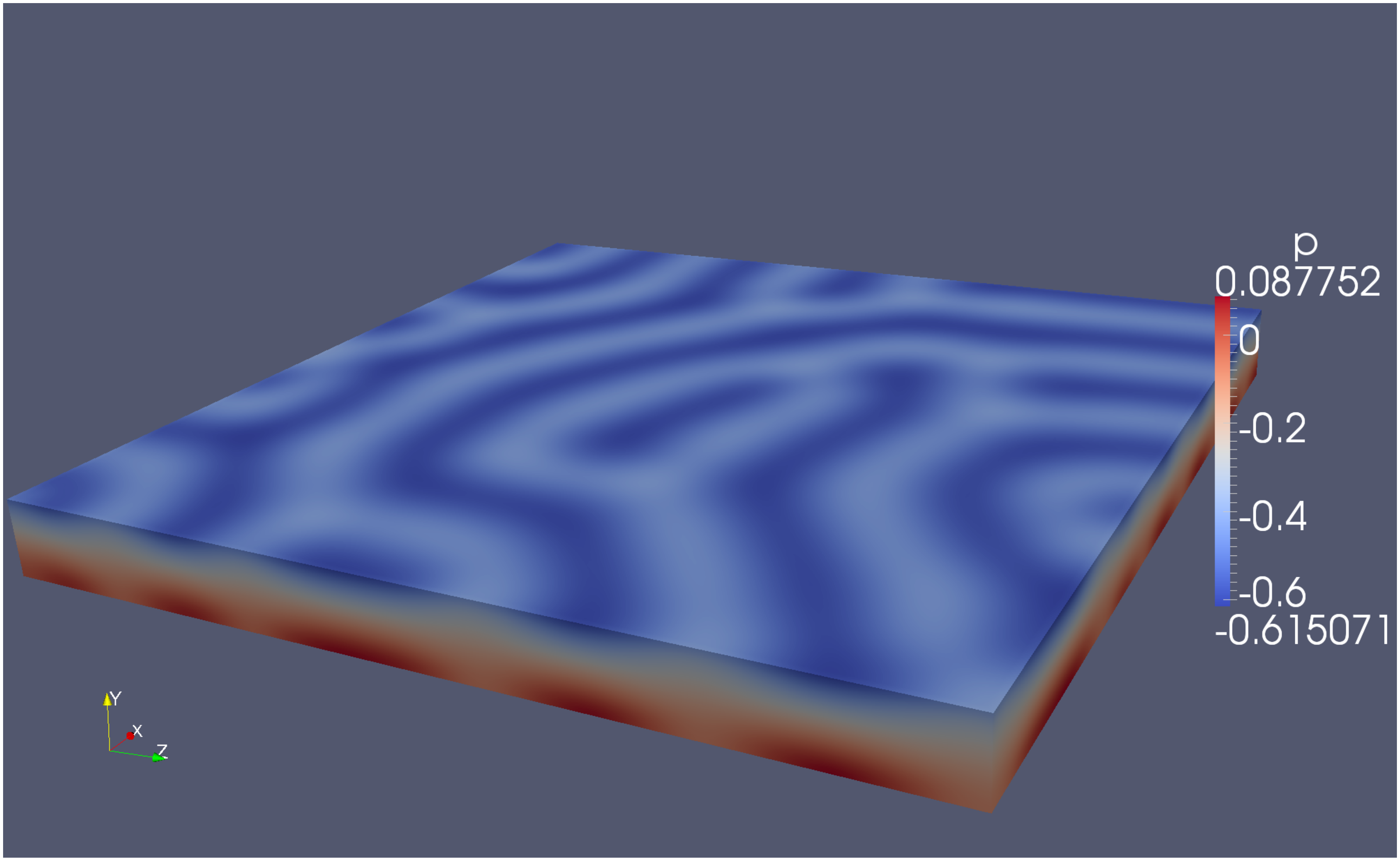
with the dimensionless porous Rayleigh number defined as

$$\mathsf{Ra} = \frac{\rho_0^2 c g \beta (T_1 - T_0) \kappa L}{\mu k}$$

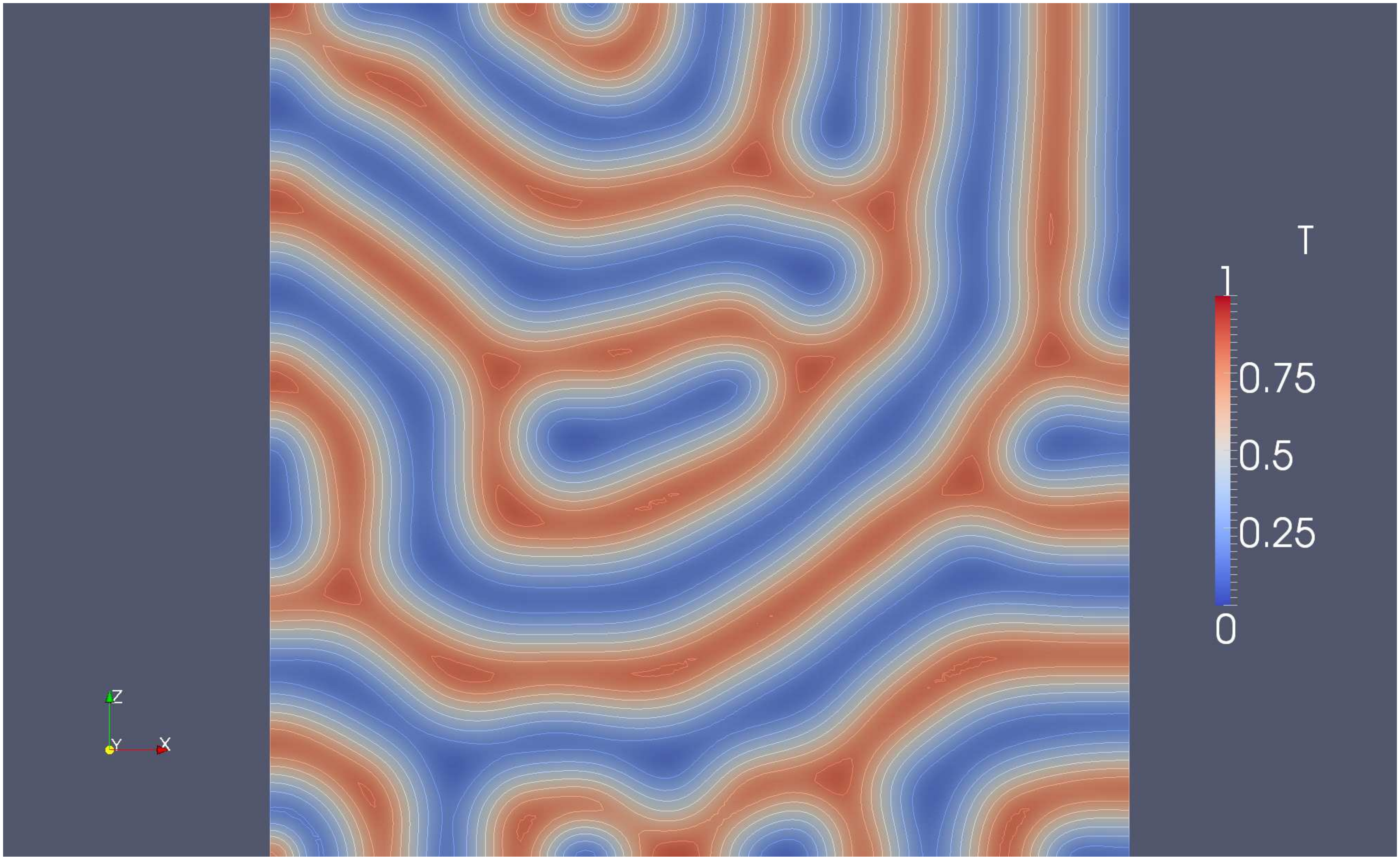
A customized solver code

```
    while (runTime.loop())
    {
        Info<< "Time = " << runTime.timeName() << nl << endl;

#       include "readPISOControls.H"
#       include "CourantNo.H"

        for (int nonOrth=0; nonOrth<=nNonOrthCorr; nonOrth++)
        {
            fvScalarMatrix pEqn
            (
                // Darcy equation for porous flow
                fvm::laplacian(kappa / nu, p) + kappa / nu * fvc::div(gflux, rhok)
            );

            // Set reference pressure and solve Darcy equation
            pEqn.setReference(pRefCell, pRefValue);
            pEqn.solve();

            // Update velocity field and flux
            U = -kappa / nu * (fvc::grad(p) + rhok * g);
            phi = fvc::interpolate(U) & mesh.Sf();

            solve
            (
                // Solve heat transport equation
                fvm::ddt(T) + fvm::div(phi, T) - fvm::laplacian(nu / Pr, T)
            );

            // Update kinematic density, based on Boussinesq approximation
            rhok = 1.0 - beta*(T - TRef);
        }

        runTime.write();
    }
```

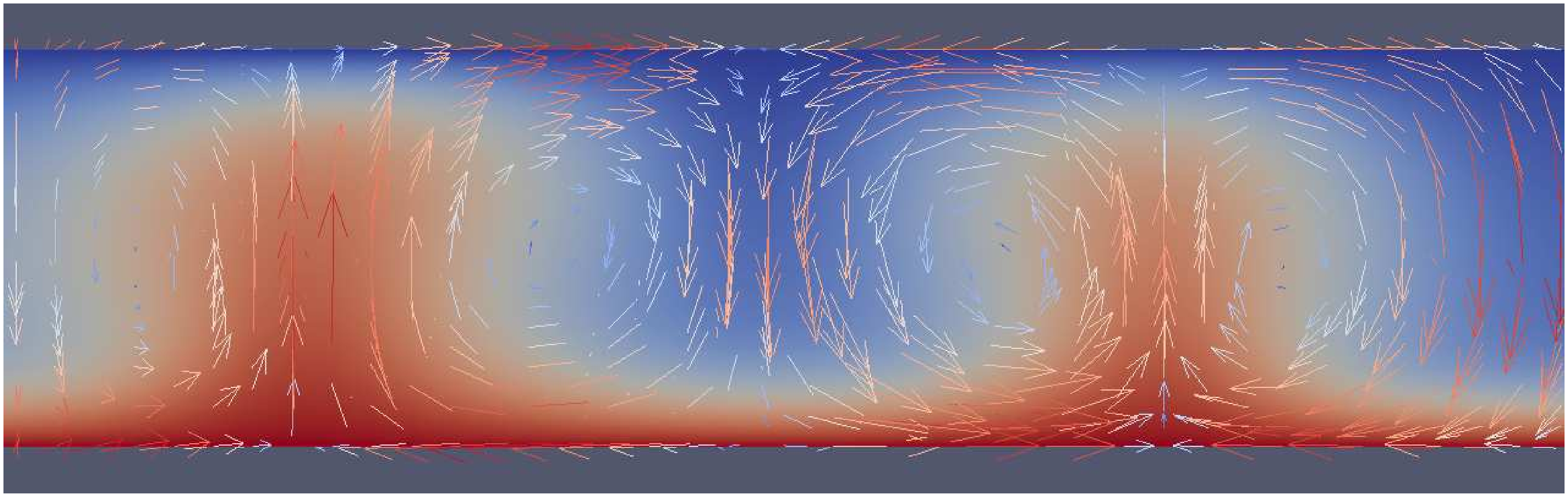
Results for Ra $= 100$, pressure



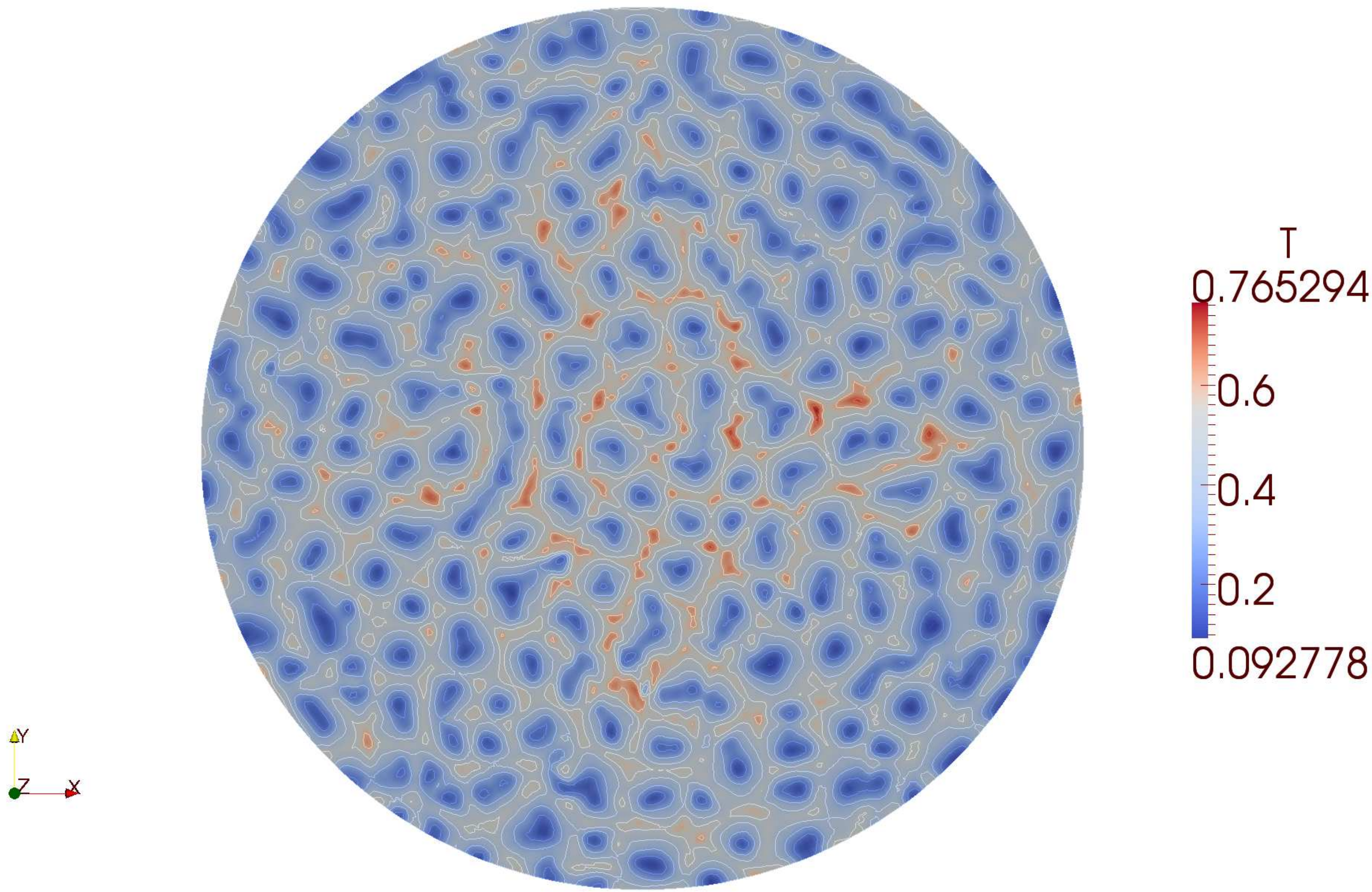
Results for Ra $= 100$, temperature



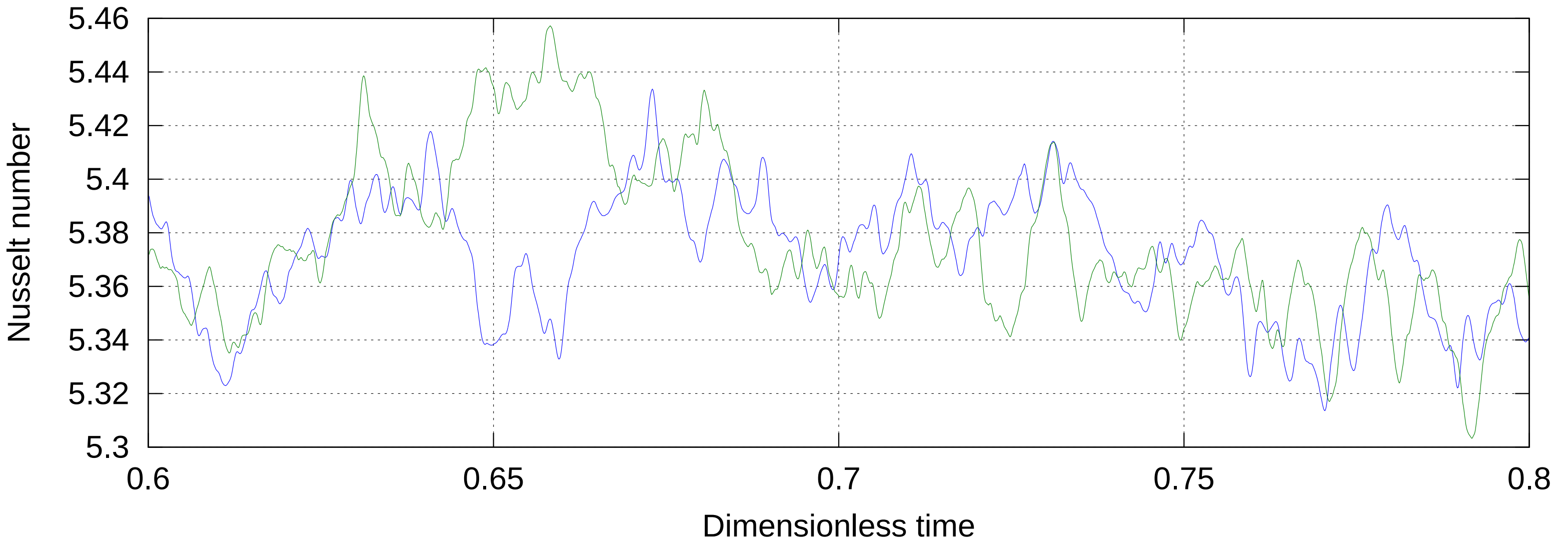
Results for Ra $= 100$, continued

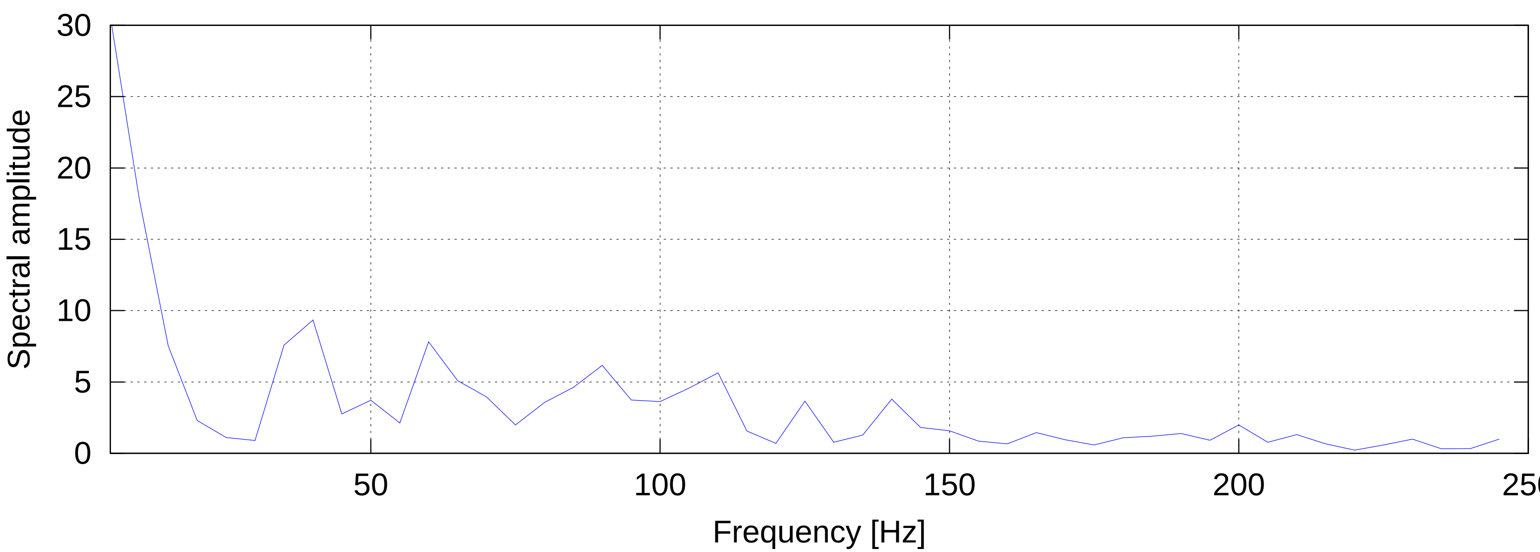


Results for Ra $= 500$, temperature



Dimensionless flux from top and bottom





Results for two dimensional cases